A NEW SCHEME TO TERMINATE ALL TRELLIS OF TURBO-DECODER FOR VARIABLE BLOCK LENGTH

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ABSTRACT

This paper presents a new method of block decoding with turbo codes, where the termination of all the used Soft-in-Soft-out decoders is possible. This leads to an improvement in the performance of these codes by the channel coding. The flattening effect at smaller bit error rates compared to the results with termination of single decoder is also suppressed. We also present a way to select polynomials for turbo codes. Finally, we discuss the achieved simulation results in AWGN and 60-GHz multipath channels.

1. INTRODUCTION

In many systems like GSM (Global System for Mobile Communications), DCS (Digital Cellular System), MEDIAN[1] etc. the digital data is transmitted in frames. To encode, transmit and decode the frames independently from one and other, mostly block codes are preferred. After the introduction of the turbo codes in [1], many results for the block decoding using these codes were presented in [2], [3]. Due to the application of the 'Recursive Systematic Codes (RSC)' the difficulties in terminating the trellis of the decoders were also pointed out. With the presentation of the Turbo-Block-Codes in [4] a hurdle of terminating single trellis is overcome. In the next sections we present a new method of turbo block decoding, where the termination of all the Soft-in-Soft-out Viterbi (SOVA) decoders is possible by using the tail bits and the polynomial division property of the RSC codes which was illustrated in [4]. This provides an extra redundancy information to the decoders which leads to an improvement in the performance of the correction capacity of these codes. Through terminating both the SOVA decoders of an iteration, the early flattening effect which results by keeping the trellis of the decoders open is also suppressed, but not completely removed. We also enlight the use of primitive polynomials and the flexibility of the cell size for a fixed code polynomial and given interleaver size. Finally a short discussion is given on the achieved simulation results.

2. SELECTION OF POLYNOMIALS FOR TURBO-CODES

The selection of good polynomial combinations play an important role by convolutional codes. Here we give a concise description of the polynomial division property for the selection of a recursive polynomial. For details please refer to [4]. All the considered polynomials $G_1(D)$ for the recursion of the encoder have a particular polynomial $P(D)$ which satisfy the equation

$$P(D) = f(D) * G_1(D) = 1 + D^l. \qquad (1)$$

We define $P(D)$ as the reset polynomial and l as the grade of it. The division of $P(D)$ by $G_1(D)$ is aliquot. If $G_1(D)$ divides $1 + D^l$ without any remainder, then it also divides $1 + D^{n*l}$, where 'n' is a natural number greater than 0. By encoding a sequence related to $1 + D^{n*l}$ through a RSC encoder starting at the zero state, the encoder will be driven back to the zero state after the end of the sequence. This property along with the linearity feature is used to terminate the trellis of the second decoder of an iteration by turbo codes.

If the coderate of the encoder is restricted to '½', the selection of good polynomial combinations can be done with the equation

$$G_1(D) + G_2(D) = D * H(D), \qquad (2)$$

where the grades of $G_1(D)$ and $G_2(D)$ are equal to the memory of the encoder 'm' and that of $H(D)$ is 'm-2'. $G_1(D)$ and $G_2(D)$ differ in the coefficients D and D^{m-1}. All the considered polynomials include the term '1'. This class of codes possess a large free distance and does not lead to catastrophic codes [5]. These codes also include some of the well know primitive polynomials. For a primitive polynomial there exists a $P(D)$ with $l = 2^m - 1$.

3. MODIFIED TURBO-ENCODER

The modified turbo encoder uses tail bits to drive the first encoder to zero state and the above

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[1] European Research Project AC006 in the framework of the ACTS program of the 4th Research program of the European Commision

mentioned property along with the linearity character of the convolutional codes for the termination of second trellis. Figure 1 depicts the modified encoder.

The information data '$\overline{d}$' of '*N*'-bits, where *N* represents the size of a block, is first passed through the RSC encoder and at the same time fed into an interleaver of size equal to $(N + N_t)$ with 'N_t' denoting the number of tail bits, i.e. size of '$\overline{t}$'. The interleaver should assure that each bit *d(t)* at the input should be equal to the output bit *d(t+x)*, with $x = (N + N_t + n * l)$. A logic circuit inspects the present state of the RSC encoder and generates the respective bit, with which the encoder is driven to the zero state in short time. This logical circuit can also be substituted with the modification advocated in [6]. After the last information bit has been passed through the coder, the 'S_1' switch selects N_t tail bits which depends on the memory of the encoder. If the sum of the data bits and the tail bits is not a multiple of the grade of reset polynomial, then 'N_0' number of zero bits '$\overline{0}$' are introduced into the data input of the RSC encoder by switching 'S_3'.

$$\begin{aligned} N_0 &= i * l - (N + N_t) \\ &\geq 0 \end{aligned} \tag{3}$$

'*i*' is the smallest integer which satisfies the required condition. During the insertion of zero bits the clocking of the interleaver is stopped. At the same time the output of the encoder is ignored, as these bits are simply used to satisfy the polynomial division property. Once the last zero bit has been passed through the encoder the reading of the interleaver data is done by selecting the 'S_2' switch. Through this modified method the number *N* is not fixed to a value which is a multiple of the grade of the reset polynomial. It is possible to encode all the block lengths smaller than *N*. Through expanding the size of the interleaver the maximum number of bits per block can be increased. This type of encoding is similar to the shortened Reed-Solomon-Codes. Since the blocks are to be independent of each other, a continuous encoding is not possible.

For instance, a RSC encoder with the octal representation of the polynomials {13,15} is considered. The recursion polynomial is 13. The grade *l* of the reset polynomial is 7 and requires N_t=3 tail bits. If *N* is 440 then the total size of the interleaver would be 443 with 22 rows and 21 columns where the last row consists of only two elements. The interleaver is filled with the data in rows and the reading is done column by column. This makes sure that each input bit of the interleaver is written out after (443+7**n*). If a search for an optimal interleaver is performed, then the shuffling of the order of reading is allowed only within a column. In the considered example there is a need of 5 zero bits so that the size of input frames of the encoder is a multiple of its reset polynomial grade 7. Now the systematic part '*X*' consists of the data bits and the tail bits. The redundancy part '*Y*' is punctured to obtain the required code rate.

4. DECODING OF THE TURBO-BLOCK-CODES

The construction of an iteration of the turbo decoder is similar to that in [1]. Each iteration consists of two MAP [7] or SOVA [8] decoders in serial to realize the soft input and soft output decoding. Here we consider only the SOVA algorithm as the realization of this algorithm is less complex than that of the symbol-by-symbol MAP algorithm at a cost of small loss in the correction capacity. Both the decoders are separated by an interleaver of same size as used by the modified encoder. After the reception of a block, decoding of it is done. In decoding the modified turbo block codes, an extra information is availed. This information says that all the decoders start in zero state and terminate in the zero state. The termination of the first decoder is achieved with the tail bits. The second decoder's trellis is terminated using the polynomial division and linearity properties. This art of decoding minimizes the burst errors at the end of a block which occur if the applied Viterbi algorithm selects a wrong path. This is mostly the case in block decoding with Viterbi algorithm as the observation length of the last received bits is less than thrice the memory length. Since the decoding starts in the zero state, a metric value greater than that of the other state metrics is provided to the zero state while initializing the values.

5. SIMULATION RESULTS

For the selection of good polynomial combinations with memory lengths 3 and 4, a continuos turbo decoder with two iterations, BPSK modulation and AWGN channel are considered. The best simulation results of all the combinations which satisfy equation 2 are plotted in figure 2.

The first polynomial with the octal representation in the brackets is considered as the recursion polynomial. It is seen for both the memory lengths 3 and 4, the best combinations include primitive polynomials as the recursion.

To test the efficiency of the new modified block decoding of turbo codes simulations were carried out for block length of 440 bits which is equal to an ATM cell with two extra bytes for frame informations in case of wireless transmission like in MEDIAN. Figure 3 shows the comparison of continuous and block decoding for BPSK modulation after two decoder iterations. The output of the RSC{13, 15} encoder is punctured in order to acquire code rates 1/2 and 5/7.

'TC1[440;880]' represents the continuous Viterbi decoding with SOVA, truncation path length 50 and code rate '1/2'. 'TC2[440;883]' shows the new modified decoding with the termination of trellis of all decoders whereas 'TC3[440;880]' represents the art of decoding where only the second trellis of an iteration is terminated. The redundancy part of the TC2 also include the 3 tail bits to terminate the trellis of the first decoder of an iteration. Before the transmission, zero bits were punctured out and introduced into the frame after receiving. This was done to make the decoding block length to be 448 and the observation length 56. This made it possible to flush the decoded informations after 8 loops. The independence of the blocks is assured by the decoding process. Due to the flushing out process, the last soft informations of a block delivered by the SOVA decoder were not as relevant as the previous informations since the observation length was less than thrice the memory length of the encoder. This leads to a degradation in the correction performance compared to the continuous decoding where each and every bit has the observation length of 50. It is also seen that the early flattening effect around bit error rate of 10^{-5}, which occurs by keeping the trellis of the first decoder open, was also suppressed by terminating both the decoders, but not completely removed. 'TC4[440;616]' represents the continuous Viterbi decoding for code rate '5/7'. 'TC5[440;619]' stands for the art of decoding, where all the trellis of the decoders are terminated whereas 'TC6[440;616]' shows the results achieved by decoding where the trellis of 2nd decoders of iterations were terminated.

The application of these codes is also tested for a wireless 312 subcarrier OFDM system with AWGN and simplified LOS model of the 60 GHz indoor radio channel presented in [9]. The time and frequency synchronisations are done for every frame with the help of a reference symbol. Each frame consists of 64 symbols including reference symbol. Figure 4 depicts the achieved results for DQPSK modulation and AWGN and LOS channels with turbo block codes (two iterations) and Reed-Solomon Coding schemes. All the code rates are nearly equal to '5/7'.

TC7[440;619] and TC8[440;616] represent the AWGN channel with termination of all decoders and termination of second decoder respectively. RS1[440;616] shows the BER for AWGN channel
At BER of $2*10^{-5}$, the new turbo block codes shows a gain of 2.8 dB on signal-to-noise ratio compared with Reed-Solomon codes.
TC9[440;619] and TC10[440;616] represent the simulation results for a LOS channel with termination of all decoders and termination of second decoder respectively. It is seen that for small BER the flattening effect in case of LOS channel is reduced drastically. RS2[440;616] denotes the achieved results for Reed-Solomon code. If the BER of $2*10^{-5}$ is considered, the new turbo block codes shows a gain of 2.5 dB on signal-to-noise ratio for LOS channel model compared with Reed-Solomon codes.

6. CONCLUSIONS

We have shown a new art of turbo block decoding, where the termination of all the SOVA decoders results in an improvement of the correction capacity of the decoder at high SNR values by suppressing the early flattening effect. This art of decoding allows to keep the block length variable for a given code polynomial and interleaver size. We have also presented a way to select good generator polynomials for this coding scheme.

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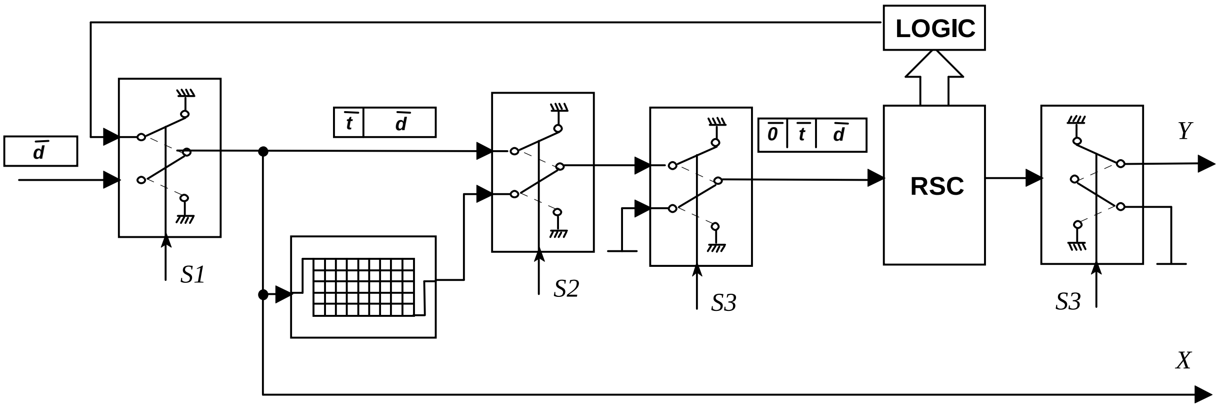

Figure 1: *Modified Turbo Block Encode*r.

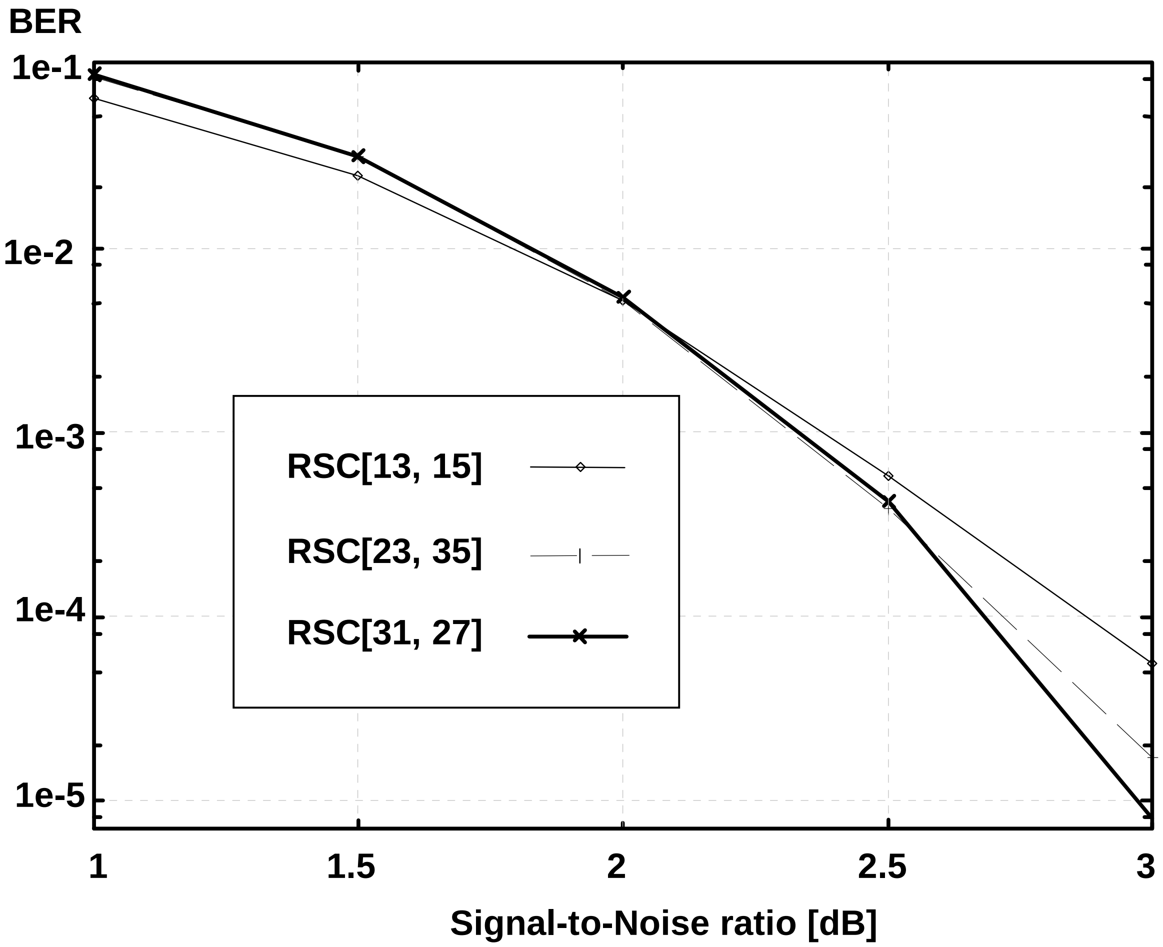

Figure 2: *Comparison between different polynomial combinations.*

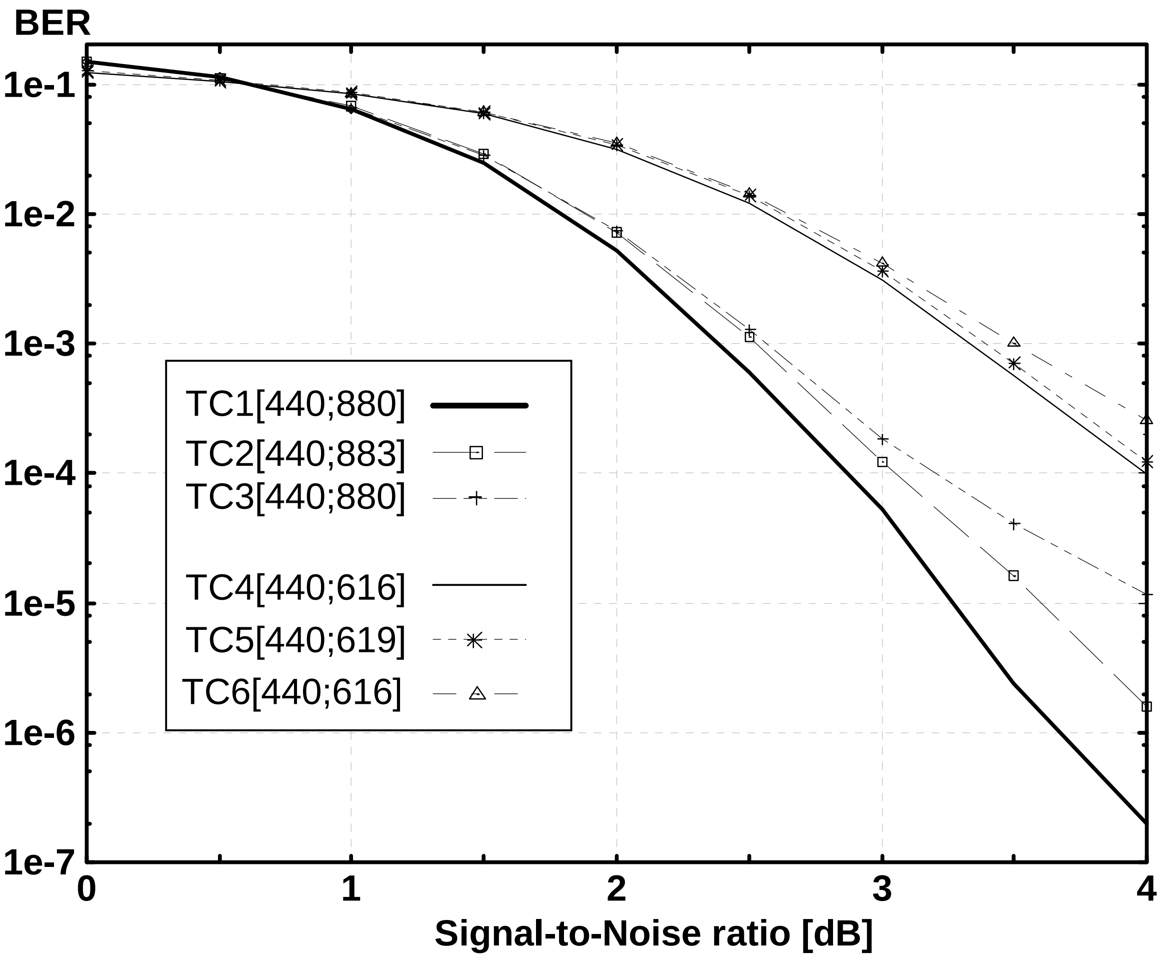

Figure 3: *Comparison of continuous and Turbo Block decoding.*

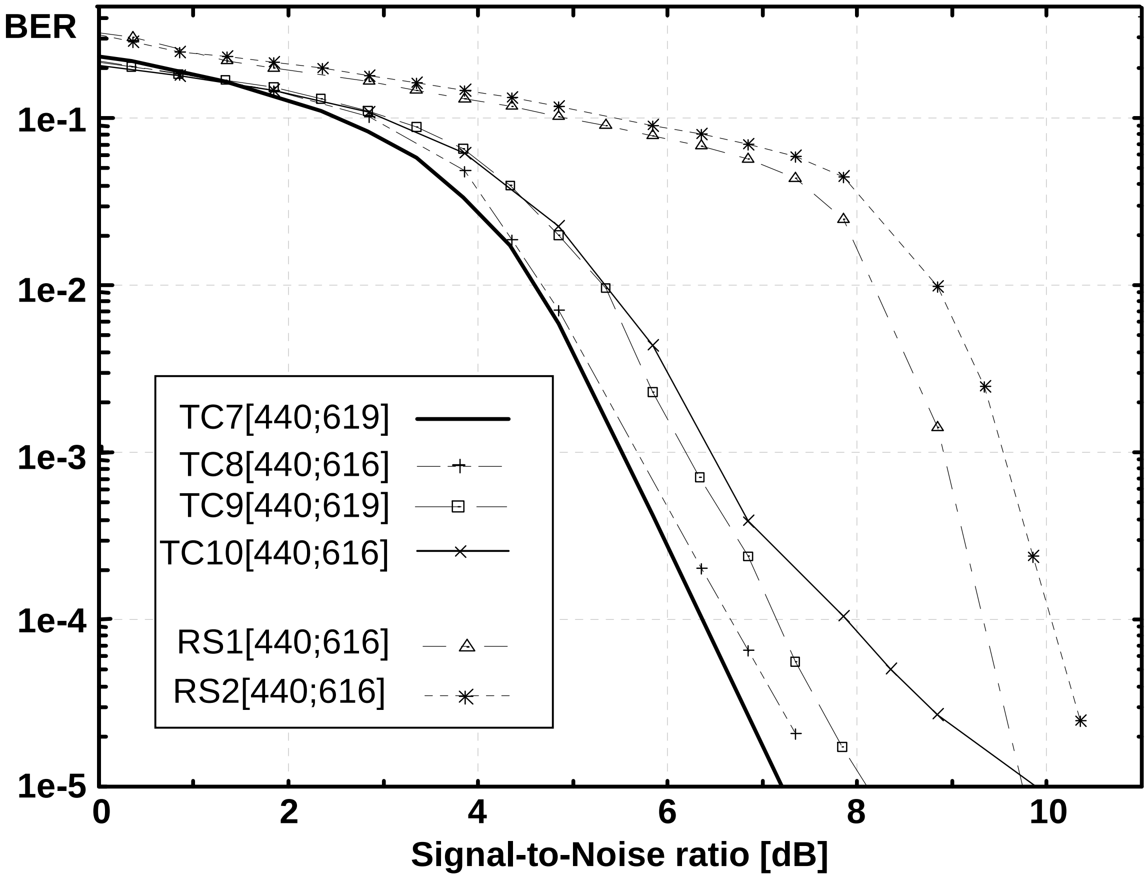

Figure 4: *Comparison of TURBO-Codes with Reed-Solomon codes for 312 subcarrier DQPSK-OFDM system in AWGN and LOS with both time and frequency error correction.*